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# A life-cycle approach to the intertemporal elasticity of substitution

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**Abstract**

We construct a three-period model spanning 30 years of an optimizing consumer's life. Exploiting the first-order conditions, we derive expressions for the intertemporal elasticity of substitution (IES) that allow for different utility specifications; the case of isoelastic utility is a special case. We fit US household data on income, consumption, and net worth to the IES expressions to obtain point estimates of the IES. We also construct 95 percent confidence intervals, based on 10,000 simulated observations. Our evidence suggests that the value of the IES is likely between 0.2 and 0.8.


*JEL classification:* E21

*Keywords:* Intertemporal elasticity of substitution

## 1. Introduction

The intertemporal elasticity of substitution (IES) measures the extent to which an increase in the interest rate induces consumers to substitute future consumption for present consumption. The emergence in recent years of an extensive IES literature reflects the widespread recognition that the magnitude of this elasticity is fundamental to many important issues in macroeconomics. In large part, contributions to that literature have followed Hall's (1988) approach, in which (a) the representative consumer's utility func-

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tion is assumed to be of the isoelastic type and (b) aggregate consumption data are used in estimating the value of the IES. In this paper we take a different approach. In particular, we develop a framework that is not dependent upon the assumption of isoelastic utility. Furthermore, as an alternative to aggregate consumption data, we use survey data on consumption, income, and net worth.

Hall's approach, in which the growth rate of aggregate real consumption of nondurable goods is regressed on the expected real interest rate, typifies attempts, to date, to estimate the IES. Employing various techniques and data sets, Hall produced several estimates of the IES, all of which were small in absolute value and not significantly different from zero. On the basis of his findings, Hall concluded that consumption growth remains close to its average value, irrespective of the level of interest rates. The IES, he asserts, "may even be zero and is probably not above 0.2" (350). Using a model similar to Hall's, but incorporating estimation methods that are widely regarded as superior to those used by Hall, Hansen and Singleton (1996) produced negative estimates of the IES.

Several challenges to Hall's basic approach have been mounted, with varying degrees of success. One challenge stems from the notion that, rather than strictly conforming to the outcome of intertemporal optimization problems, consumption spending reflects the behavior of liquidity-constrained and "rule-of-thumb" consumers. Campbell and Mankiw (1989), for example, modified the standard Euler equation by allowing a fraction of consumers to follow a rule-of-thumb that calls for them to consume all of their income in each period.[1] Nevertheless, Campbell and Mankiw's findings corroborate Hall's (1988), as they are unable to reject the hypothesis of an IES value of zero.

Patterson and Pesaran (1992) examined the robustness of Hall (1988) and Campbell and Mankiw's (1989) findings to their assumption of a first-order moving average. Estimating the moving average term using an instrumental variable method and allowing for rule-of-thumb consumers, Patterson and Pesaran found continued support for the proposition that the IES is not significantly different from zero. However, Beaudry and Van Wincoop (1996) argued that estimates of the IES become imprecise when rule-of-thumb consumers are accounted for. They found (1996, 496) that for aggregate US data on consumption of nondurable goods, "almost any value between 0 and 1.5 cannot be rejected." Using a panel of state-level data they found the IES to be significantly different from zero and close to 1. Furthermore, Runkle (1991) found no evidence that consumers face liquidity constraints, and he estimated the IES to be 0.45 and statistically different from zero.[2]

A second challenge to Hall's approach pertains to its emphasis on consumption of nondurable goods. Ogaki and Reinhart (1998) argue that the exclusion of spending on durable goods – as is the case in Hall (1988) and Hansen and Singleton (1996) – biases the estimates of the IES. Noting that the real interest rate influences the user cost of the purchase of a durable good, Ogaki and Reinhart maintain that an increase in the interest rate causes consumers to substitute current consumption away from durable goods, toward nondurable goods. The effect, they argue, is to reduce future growth in nondurable consumption, relative to the case of no change in the user cost. Allowing for non-separable preferences in consumption of durable and nondurable goods, the authors estimated the value of the IES

[1] Campbell and Mankiw (1989) find that 45% of consumers follow the "rule-of-thumb."

[2] In contrast to Runkle's findings, Jappelli (1990) found that 19% of US consumers are liquidity-constrained.

to be between 0.33 and 0.45 and statistically different from zero.[3] Using the same methods, applied to data for Japan, Fuse (2004) obtained IES estimates ranging from 3.7 to 4.4.

The assertion that heterogeneity among consumers should be accounted for constitutes a third challenge to Hall's approach. Attanasio and Weber (1995) posited a set of preferences that controls for the effects of changes in demographics and labor-supply behavior over the life-cycle. By tracking cohorts of consumers over time, they provided evidence in support of the life-cycle hypothesis, which asserts that income, consumption, and family size are hump-shaped. Using data from the Consumer Expenditure Survey, Attanasio and Weber estimated the IES to be 0.56 for their entire sample, and 0.67 for a shorter time period. Hamori (1996) also captured heterogeneity, through differences in preferences among consumers with different incomes. Hamori's results indicate that for Japanese households the IES ranges from 1.0 to 1.4, and that the IES decreases with income.

In this paper, we employ a three-period model of discounted utility maximization, subject to an intertemporal budget constraint, that covers a 30-year span in the life of a representative household. From the first-order conditions, we derive equations that relate the IES to the household's consumption, income, and net worth over the three-periods. Using household data on those magnitudes, we fit the mean values of the data to the model to calculate point estimates of the IES for two cohorts of households – over the relevant time period, Cohort I heads of household progress from age from 45 to age 74, while Cohort II heads of household progress from age 35 to age 64. To supplement our point estimates, we construct confidence intervals to allow for sampling error in the survey data.

Our approach follows up on the second and third challenges to the more traditional analysis. Our consumption data incorporate spending on both durable and nondurable goods. In addition, our analysis captures consumer heterogeneity in a manner similar to that of Attanasio and Weber (1995). Furthermore, our formulation of the consumer's (household's) optimization problem is relatively general; rather than assuming a particular utility function at the outset, the formulation allows for a variety of utility functions. Finally, our life-cycle framework allows us to estimate an array of intertemporal substitution elasticities. Specifically, we present point and interval estimates of *cross*-period elasticities in addition to estimates of *intra*-period elasticities. The existing literature's neglect of the cross-period elasticities, coupled with the usual assumption of a constant-elasticity utility function, leads most authors to refer to "the" IES, when in fact there is more than one elasticity to identify and estimate.

The paper is structured as follows. In Section 2, the household's three-period optimization problem is formulated and solved, and expressions for the various substitution elasticities are derived. In Section 3, we present the data that are used to estimate the values of the substitution elasticities. Point estimates and interval estimates of the elasticities are presented and discussed in Section 4, and conclusions follow in Section 5.

## 2. The household's problem

We assume the household formulates and carries out a plan that spans three 10-year periods. The household's objective is to maximize a discounted sum of utilities,

[3] It should be noted that, in contrast to Ogaki and Reinhart's (1998) use of a utility function that is non-separable between durable and nondurable goods, our single-good model does not incorporate this non-separability feature.

$\sum_{t=1}^{3}\beta^{t-1}U(c_t)$, where $c_t$ denotes household consumption in period $t$. The period utility function, $U$, is strictly increasing, twice- differentiable, and strictly concave, with the subjective discount factor, $\beta$, satisfying $0 < \beta \leqslant 1$. For simplicity we assume the household is blessed with perfect foresight.

In each period, the household can lend or borrow on a one-period (10-year) basis. On loans that are made at the beginning of period $t$, interest is applied at the beginning of period $t+1$. We use $y_t$ to denote income for period $t$, $a_t$ to denote end-of-period-$t$ assets (or net worth), and $R_t$ to denote the gross interest rate on loans made at the beginning of period $t$. All variables, including the gross interest rate, are in real terms. Using $a_0$ to represent initial assets, we have

$$a_{t+1} = R_{t+1}a_t + y_{t+1} - c_{t+1} \quad (t = 0, 1, 2). \tag{1}$$

Using (1), it is straightforward to derive the household's three-period budget constraint:

$$R_1R_2R_3a_0 + R_2R_3(y_1 - c_1) + R_3(y_2 - c_2) + y_3 - c_3 - a_3 = 0. \tag{2}$$

Formally, the household's problem is to maximize $\sum_{t=1}^{3}\beta^{t-1}U(c_t)$ subject to (2), where the values of initial assets, $a_0$, and terminal assets, $a_3$, are regarded as exogenous. The pair of first-order (Euler) conditions,

$$U'(c_t) = \beta R_{t+1}U'(c_{t+1}) \quad (t = 1, 2) \tag{3}$$

together with (2), are necessary and sufficient for a solution due to the strict concavity of $U$.

### 2.1. Elasticities of substitution

To derive the intertemporal elasticities of substitution, we begin by totally differentiating (2), allowing the values of consumption and the interest rates to vary:

$$\begin{aligned} &R_2R_3a_0\,\mathrm{d}R_1 + (R_1a_0 + y_1 - c_1)R_3\,\mathrm{d}R_2 + [(R_1a_0 + y_1 - c_1)R_2 + y_2 - c_2]\mathrm{d}R_3 \\ &\quad - R_2R_3\,\mathrm{d}c_1 - R_3\,\mathrm{d}c_2 - \mathrm{d}c_3 = 0. \end{aligned} \tag{4}$$

Define $\sigma_t = -c_tU''(c_t)/U'(c_t)$ for $t = 1, 2, 3$, and note that $\sigma$ is the absolute value of the elasticity of marginal utility with respect to consumption; that is, $\sigma_t \equiv |(\mathrm{d}U'(c_t)/\mathrm{d}c_t)\cdot(c_t/U'(c_t))|$. Totally differentiating (3) and employing the definition of $\sigma$, we have

$$\mathrm{d}R_{t+1} + R_{t+1}\left(\frac{\sigma_t}{c_t}\mathrm{d}c_t - \frac{\sigma_{t+1}}{c_{t+1}}\mathrm{d}c_{t+1}\right) = 0 \quad (t = 1, 2). \tag{5}$$

Define $x_{t+1} = c_{t+1}/c_t$ for $t = 1,2$. Application of the quotient rule for derivatives yields

$$dx_{t+1} - \frac{c_t\,\mathrm{d}c_{t+1} - c_{t+1}\,\mathrm{d}c_t}{c_t^2} = 0 \quad (t = 1, 2). \tag{6}$$

We can regard $\mathrm{d}c_1$, $\mathrm{d}c_2$, $\mathrm{d}c_3$, $\mathrm{d}x_2$, and $\mathrm{d}x_3$ as unknowns and solve using the five Eqs. (4)–(6).

Our focus is on the intertemporal consumption ratios, $x_2$ and $x_3$, and the extent to which those ratios are sensitive (in a comparative-statics sense) to changes in interest rates; that is, our primary interest is in measuring the extent to which consumers engage in intertemporal substitution in response to interest rate changes. The concept upon which we base our particular notion of the intertemporal elasticity of substitution is known in the literature as the *direct* elasticity of substitution, and typically it is defined as the percent change in a certain consumption ratio relative to the percent change in the corresponding marginal rate of substitution (MRS).[4] For example, consider the consumption ratio $x_2(\equiv c_2/c_1)$ and note that (3) gives an expression for the MRS, at the optimum, between consumption in periods 1 and 2: $U'(c_1)/U'(c_2) = \beta R_2$. Noting that $\beta$ is constant, the elasticity of $x_2$ with respect to $R_2$ therefore can be written as

$$\varepsilon_{22} \equiv \frac{\partial(c_2/c_1)}{\partial[U'(c_1)/U'(c_2)]} \cdot \frac{U'(c_1)/U'(c_2)}{c_2/c_1} = \frac{\partial x_2}{\partial R_2} \cdot \frac{R_2}{x_2}. \tag{7}$$

We will refer to the elasticity in (7) as an *intra-period* elasticity since it measures the response of the consumption ratio $c_2/c_1$ to the interest rate that spans periods 1 and 2. In the same vein, we have a second intra-period elasticity, namely, the one that measures the response of the consumption ratio $c_3/c_2$ to the interest rate that spans periods 2 and 3: $\varepsilon_{33} \equiv (\partial x_3/\partial R_3) \cdot (R_3/x_3)$. In view of the foregoing, it seems natural to define two cross-period elasticities, $\varepsilon_{23} \equiv (\partial x_2/\partial R_3) \cdot (R_3/x_2)$ and $\varepsilon_{32} \equiv (\partial x_3/\partial R_2) \cdot (R_2/x_3)$. Now, all four of the elasticities defined thus far are directly tied to the usual notion of the direct elasticity of substitution, since from (3) it is apparent that a change in $R_2$ (respectively, $R_3$) will induce a proportionate change in $U'(c_1)/U'(c_2)$ (respectively, $U'(c_2)/U'(c_3)$). Recall that there is a third interest rate, $R_1$, which is the rate a consumer earns, during his first period, on his initial wealth. A change in $R_1$ can induce changes in the intertemporal consumption ratios, and so we define two more cross-period elasticities: $\varepsilon_{21} \equiv (\partial x_2/\partial R_1) \cdot (R_1/x_2)$ and $\varepsilon_{31} \equiv (\partial x_3/\partial R_1) \cdot (R_1/x_3)$. It should be noted, however, that $R_1$ does not appear in (3). Thus, these last two elasticities extend the notion of the direct elasticity of substitution in the sense that, unlike the other four elasticities, they cannot be defined in terms of an MRS.

In summary, we have used the model's two intertemporal consumption ratios and its three interest rates to define six intertemporal elasticities of substitution – we have $\varepsilon_{ij} \equiv (\partial x_i/\partial R_j) \cdot (R_j/x_i)$ for $i = 1, 2, 3, j = 2, 3$. Solution of (4)–(6) yields the following explicit expressions:

$$\varepsilon_{22} = \frac{(\sigma_1 - \sigma_2)\sigma_3 R_2 R_3 a_1 + \sigma_3 R_2 R_3 c_1 + \sigma_3 R_3 c_2 + \sigma_2 c_3}{\sigma_2\sigma_3 R_2 R_3 c_1 + \sigma_1\sigma_3 R_3 c_2 + \sigma_1\sigma_2 c_3}, \tag{8}$$

$$\varepsilon_{33} = \frac{(\sigma_2 - \sigma_3)\sigma_1 R_3 a_2 + \sigma_2 R_2 R_3 c_1 + \sigma_1 R_3 c_2 + \sigma_1 c_3}{\sigma_2\sigma_3 R_2 R_3 c_1 + \sigma_1\sigma_3 R_3 c_2 + \sigma_1\sigma_2 c_3}. \tag{9}$$

$$\varepsilon_{21} = \frac{(\sigma_1 - \sigma_2)\sigma_3 R_1 R_2 R_3 a_0}{\sigma_2\sigma_3 R_2 R_3 c_1 + \sigma_1\sigma_3 R_3 c_2 + \sigma_1\sigma_2 c_3}, \tag{10}$$

[4] The elasticity of substitution literature grew out of the production-function literature; see, for example, Uzawa (1962) and McFadden (1963). As noted by McLaughlin (1995), three elasticities of substitution have been identified and employed in the literature. They are the direct elasticity (used here), the Hicks–Allen elasticity, and the marginal-utility-of-wealth-constant own-price elasticity. McLaughlin shows that all three elasticities have the same value when the utility function is isoelastic. Our notion of the direct elasticity is identical to the discrete-time version of the elasticity of substitution defined in Blanchard and Fischer (1989, 40). It is also identical to the elasticity of substitution defined in Azariadis (1993, 179-80). McLaughlin (1995, 198) notes that precise definitions of the elasticity of substitution are seldom offered, and he credits Mankiw et al. (1985) and Hall (1988) for their explicit adoption of the direct elasticity.

$$\varepsilon_{23} = \frac{(\sigma_1 - \sigma_2)(\sigma_3 R_3 a_2 - c_3)}{\sigma_2\sigma_3 R_2 R_3 c_1 + \sigma_1\sigma_3 R_3 c_2 + \sigma_1\sigma_2 c_3}, \tag{11}$$

$$\varepsilon_{31} = \frac{(\sigma_2 - \sigma_3)\sigma_1 R_1 R_2 R_3 a_0}{\sigma_2\sigma_3 R_2 R_3 c_1 + \sigma_1\sigma_3 R_3 c_2 + \sigma_1\sigma_2 c_3}, \tag{12}$$

$$\varepsilon_{32} = \frac{R_2 R_3(\sigma_2 - \sigma_3)(\sigma_1 a_1 + c_1)}{\sigma_2\sigma_3 R_2 R_3 c_1 + \sigma_1\sigma_3 R_3 c_2 + \sigma_1\sigma_2 c_3}. \tag{13}$$

As is evident from (8)–(13), each of the substitution elasticities can be expressed in terms of the values of consumption, elasticities of marginal utility, interest rates, and assets. It is certainly worth noting that, for the constant-elasticity utility function, $\sigma_1 = \sigma_2 = \sigma_3$. Denoting this common value by sigma, the intra-period elasticities reduce, for the constant-elasticity function, to $\varepsilon_{22} = \varepsilon_{33} = 1/\sigma$ and each of the cross-period elasticities is equal to zero.

### 2.2. Two particular utility functions

In subsequent sections of the paper we will apply our data to two particular forms of the utility function. Both forms are used extensively in the financial-economics literature; see, for example, Huang and Litzenberger (1988). Next, we present and discuss those two functions, the first of which is the negative-exponential utility function[5]:

$$U(c_t) = -\frac{1}{\theta} e^{-\theta c_t} \quad (\theta > 0) \tag{14}$$

for which the first-order conditions (3) can be written as

$$\theta(c_{t+1} - c_t) = \ln(\beta R_{t+1}) \quad (t = 1, 2). \tag{15}$$

The optimal consumption values can be expressed as

$$c_1^* = \frac{\theta K - (1 + R_3)\ln(\beta R_2) - \ln(\beta R_3)}{\Delta}, \tag{16}$$

$$c_2^* = \frac{\theta K + R_2 R_3 \ln(\beta R_2) - \ln(\beta R_3)}{\Delta}, \tag{17}$$

$$c_3^* = \frac{\theta K + R_2 R_3 \ln(\beta R_2) + R_3(1 + R_2)\ln(\beta R_3)}{\Delta}, \tag{18}$$

where $K \equiv R_1 R_2 R_3 a_0 + R_2 R_3 y_1 + R_3 y_2 + y_3 - a_3$ and $\Delta \equiv \theta(1 + R_2 R_3 + R_3)$. For this function, note that $\sigma_t = \theta c_t$.

The second utility function with which we will be concerned is the extended power utility function[6]:

$$U(c_t) = \frac{1}{(1-\gamma)}(\alpha + c_t)^{1-\gamma} \quad (0 < \gamma < 1 \text{ or } \gamma > 1, c_t > -\alpha) \tag{19}$$

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5 The negative-exponential function features a constant value, $\theta$, of the concavity measure $-U''(c)/U'(c)$, which, in the uncertainty literature, is the measure of absolute risk aversion.

6 The *extended* power utility function generalizes the *narrow* power utility function, $U(c) = c^{1-\gamma}/(1-\gamma)$, which is widely-employed in many branches of the literature, including, notably, the IES literature. In particular, the extended function reduces to the narrow function when $\alpha = 0$. The function exhibits constant elasticity of marginal utility (constant relative risk aversion in the uncertainty literature) only in the "narrow" case.

for which the first-order conditions (3) amount to

$$\alpha + c_{t+1} = (\beta R_{t+1})^{\frac{1}{\gamma}}(\alpha + c_t) \quad (t = 1, 2). \tag{20}$$

The optimal consumption values can be written as

$$c_1^{**} = \frac{K + \alpha R_3[1 - (\beta R_2)^{\frac{1}{\gamma}}] + \alpha[1 - (\beta^2 R_2 R_3)^{\frac{1}{\gamma}}]}{\Delta'}, \tag{21}$$

$$c_2^{**} = \frac{(\beta R_2)^{\frac{1}{\gamma}} K - \alpha R_2 R_3[1 - (\beta R_2)^{\frac{1}{\gamma}}] + \alpha(\beta R_2)^{\frac{1}{\gamma}}[1 - (\beta R_3)^{\frac{1}{\gamma}}]}{\Delta'}, \tag{22}$$

$$c_3^{**} = \frac{(\beta^2 R_2 R_3)^{\frac{1}{\gamma}} K - \alpha R_3(\beta R_2)^{\frac{1}{\gamma}}[1 - (\beta R_3)^{\frac{1}{\gamma}}] - \alpha R_2 R_3[1 - (\beta^2 R_2 R_3)^{\frac{1}{\gamma}}]}{\Delta'}, \tag{23}$$

where $K$ is as defined above and $\Delta' \equiv R_2R_3 + (\beta R_2)^{\frac{1}{\gamma}} R_3 + (\beta^2 R_2 R_3)^{\frac{1}{\gamma}}$. For this function, $\sigma_t = \gamma c_t/(\alpha + c_t)$.

## 3. The data

Our aim is to estimate the intra-period and cross-period elasticities of substitution over a 30-year span of the consumer's (household's) life-cycle. The 30-year span is divided into three 10-year periods. The model requires data for income and consumption during each period and data for net worth at the beginning and end of each period. We will estimate the elasticities by fitting the data to our optimization model and solving for the elasticity values. We do this for two cohorts of individuals: Cohort I begins the first period at age 45 and ends the third period at age 74; Cohort II begins the first period at age 35 and ends the third period at age 64. For members of each cohort, period 1 covers the years 1977–1986; period 2 covers the years 1987–1996; and period 3 covers the years 1997–2006.

Our income and consumption data are taken from the 1981, 1992, and 2002 Bureau of Labor Statistics Consumer Expenditure (CE) Surveys. The CE survey is a survey of annual income and consumption expenditures. It consists of independent quarterly surveys of approximately 7,500 households. To estimate income for a given cohort over a given 10-year period, we take the relevant mean annual income and multiply by 10. For example, to calculate income earned by a member of Cohort I during his first period (1977–1986), we use the mean income of the 45–54 age group in 1981 as the mean level of income earned each year by that individual during the 10-year period.[7] Using the numbers in Table 1, a household in cohort I earned $35,742 in 1981, which yields an estimated income of $357,420 during period 1 (1977–1986). Similar calculations are carried out to obtain income and consumption estimates for each cohort over each 10-year period.

To obtain estimates of beginning- and end-of-period net worth we use data from the Federal Reserve Survey of Consumer Finances (SCF).[8] The SCF is a triennial survey that

[7] We use data at the midpoint of the intervals to estimate average income and consumption, given that income and consumption tend to rise over time until the head of the household reaches his or her mid-50s, after which each declines. CE survey data are not available for 1982, so 1981 is used in its place. The survey is administered annually after 1984.

[8] While the Federal Reserve SCF provides data on household assets as well as household net worth, the net worth data are arguably more appropriate here in view of the fact that household liabilities, as well as household assets, are important components of the typical household's lifetime program.

Table 1
Mean annual household income and consumption (2002 Dollars)

| Year | Age of head of household | | | | | |
|---|---|---|---|---|---|---|
| | <35 | 35–44 | 45–54 | 55–64 | 65–74 | ⩾75 |
| 1981 *Y* | $30,942<br>(399) | **$36,172**<br>**(636)** | **$35,742**<br>**(720)** | $28,243<br>(618) | $18,157<br>(452) | $12,128<br>(453) |
| 1981 *C* | 32,408<br>(395) | **40,046**<br>**(587)** | **39,764**<br>**(658)** | 29,740<br>(469) | 20,699<br>(417) | 15,041<br>(361) |
| 1992 *Y* | 33,939<br>(437) | 41,446<br>(555) | **43,537**<br>**(725)** | **34,865**<br>**(821)** | 24,689<br>(570) | 18,937<br>(545) |
| 1992 *C* | 35,299<br>(473) | 43,532<br>(565) | **44,058**<br>**(674)** | **37,360**<br>**(720)** | 26,196<br>(485) | 20,193<br>(479) |
| 2002 *Y* | 39,684<br>(541) | 49,099<br>(614) | 49,481<br>(730) | **41,870**<br>**(693)** | **27,688**<br>**(608)** | 19,027<br>(361) |
| 2002 *C* | 38,235<br>(403) | 46,296<br>(458) | 45,691<br>(510) | **41,426**<br>**(580)** | **29,839**<br>**(491)** | 22,492<br>(398) |

Notes: $Y$ = income; $C$ = consumption.
Values in bold type are values for Cohort I and Cohort II households.
Numbers in parentheses are standard errors.
Source: 1981, 1992, and 2002 Consumer Expenditure Survey.

dates back to 1983 and contains information on the finances of approximately 4500 households. The model requires net worth values for the beginning and end of each period for each of the two cohorts. Given that net worth at the end of one-period is equal to net worth at the beginning of the next, data are needed for the years 1977, 1987, 1997, and 2007. In view of the fact that data for these years are not available from the SCF, we first examine data from the 1983, 1992, and 2001 surveys; see Table 2.

To estimate the mean level of net worth in 1987 (which marks the end of period 1 and the beginning of period 2 for each household) for households headed by individuals in a particular age bracket, we use data from the 1983 and 1992 surveys.[9] For that set of households, we calculate the (geometric) mean annual growth rate of net worth over that nine-year interval, and then we use that mean annual growth rate to project the 1987 value of net worth. An example will serve to clarify. From Table 2, mean net worth for households headed by individuals in the 45–54 age group was $170,780 in 1983 and $335,091 in 1992. This implies a mean annual growth rate of net worth of 7.7767% for the 45–54 age group over the 1983–1992 period. Using this growth rate together with the value of net worth in 1983 allows us to project the net worth of a household headed by a person in the 45–54 age group in 1987 at $230,429. Similar calculations are made for the other age groups and are displayed in Table 3. This method is repeated to calculate net worth for each age group in 1997, using the data from the 1992 and 2001 surveys.

In the case of net worth values for 1977 and 2007, SCF data do not exist to form nine-year intervals as above. We use a longer-term growth rate of net worth to estimate these out-of-range values. For each age group we use the annualized growth rate of net worth

9 As noted in the text, 10-year intervals are not possible given the frequency of the SCF.

Table 2
Mean household net worth (2002 Dollars)

| Year | Age of head of household | | | | | |
|---|---|---|---|---|---|---|
| | <35 | 35–44 | 45–54 | 55–64 | 65–74 | ⩾75 |
| 1983 | $52,065<br>(3483) | $117,142<br>(6817) | $170,780<br>(14,912) | $234,972<br>(30,445) | $253,362<br>(24,023) | $139,948<br>(14,707) |
| 1992 | 66,181<br>(2276) | 173,658<br>(5659) | 335,091<br>(9296) | 434,701<br>(13,383) | 368,707<br>(10,651) | 276,773<br>(9,946) |
| 2001 | 95,276<br>(4961) | 256,175<br>(7131) | 485,616<br>(12,751) | 694,221<br>(20,401) | 640,606<br>(21,038) | 427,242<br>(14,395) |

Note: Numbers in parentheses are standard errors.
Source: 1983, 1992, and 2001 Survey of Consumer Finances.

over the 18-year period 1983–2001 to project net worth backward to 1977 and forward to 2007. For example, households headed by individuals in the 45–54 age group experienced a mean annual growth rate of net worth of 5.9776% between 1983 and 2001. Using this annual rate to project backward, 1977 net worth for households in this age bracket is estimated to be $120,546. Projecting forward using the same growth rate, 2007 net worth for a household in the 45–54 age bracket is estimated to be $687,984. Similar calculations are made for each age group, using the appropriate mean annual growth rate for that group.

For Cohort I the model requires net worth values at ages 45 in 1977, 55 in 1987, 65 in 1997, and 75 in 2007. The data requirements for Cohort II are similar, bearing in mind that each Cohort II member is 10 years younger than his Cohort I counterpart at each point in time. The estimates above, though, are for a range of ages in a particular year. For example, mean net worth in 1977 is $90,249 for households in the 35–44 age group, while it is $120,546 for those in the 45–54 age group. We use the arithmetic mean of these two values, $105,398, to serve as the final estimate used in the model for the net worth of a household headed by a 45-year-old in 1977. Table 4 displays the complete list of final net worth estimates used in the model. Note that, in Eqs. (1) and (2), $a_0 = \$105,398$ for a Cohort I household, $a_0 = \$66,408$ for a Cohort II household, and so on.

Finally, we use income and consumption estimates from Table 1 together with the net worth estimates in Table 4 to determine rates of return, that is, the $R$-values; see Table 5. A Cohort I head of household, for example, progressed from age 45 to age 55 during the period 1977–1986. That household's income and consumption over that period were $357,420 and $397,640, respectively. Its net worth was $105,398 at the beginning of 1977 and $269,644 at the beginning of 1987. Inserting these values into (1) for $t = 0$, we have

Table 3
Projected mean household net worth (2002 Dollars)

| Year | Age of head of household | | | | | |
|---|---|---|---|---|---|---|
| | 25–34 | 35–44 | 45–54 | 55–64 | 65–74 | ⩾75 |
| 1977 | $42,566 | $90,249 | $120,546 | $163,753 | $185,976 | $96,471 |
| 1987 | 57,923 | 139,542 | 230,429 | 308,859 | 299,336 | 189,493 |
| 1997 | 81,031 | 215,524 | 411,794 | 563,818 | 501,147 | 352,269 |
| 2007 | 116,537 | 332,514 | 687,984 | 996,150 | 872,718 | 619,789 |

Table 4
Final estimates of mean household net worth (2002 Dollars)

| Year | Age of head of household | | | | |
|---|---|---|---|---|---|
| | 35 | 45 | 55 | 65 | 75 |
| 1977 | **$66,408** | **$105,397** | $142,150 | $174,865 | $141,224 |
| 1987 | 98,733 | **184,986** | **269,644** | 304,098 | 244,414 |
| 1997 | 148,277 | 313,659 | **487,806** | **532,482** | 426,708 |
| 2007 | 224,525 | 510,249 | 842,067 | **934,434** | **746,254** |

Note: Values in bold type are estimated values of beginning– and end-of-period net worth for Cohort I and Cohort II households.

Table 5
Gross real rates of return on net worth

| Period | | Age of head of household | | | |
|---|---|---|---|---|---|
| | | 35–44 | 45–54 | 55–64 | 65–74 |
| 1977–1986 | 10-Year rate | **3.3690** | **2.9400** | 2.2446 | 1.5431 |
| | Annual rate | **1.1291** | **1.1139** | 1.0842 | 1.0443 |
| 1987–1996 | 10-Year rate | 3.3881 | **2.6652** | **2.0673** | 1.4527 |
| | Annual rate | 1.1298 | **1.1030** | **1.0753** | 1.0381 |
| 1997–2006 | 10-Year rate | 3.2521 | 2.5638 | **1.9065** | **1.4419** |
| | Annual rate | 1.1252 | 1.0987 | **1.0667** | **1.0373** |

Note: Values in bold type are estimated real rates of return for Cohort I and Cohort II households.

$269{,}644 = 105{,}398R_1 + 357{,}420 - 397{,}640$, implying $R_1 = 2.9399$. Taking the geometric mean of this 10-year rate, we obtain a gross annual rate of 1.1139, indicating the Cohort I household experienced a net annual real rate of return of 11.39% on its net worth during its first period.

Note that, for each of the three 10-year periods, rates of return decrease uniformly with the age of the head of household. This pattern is consistent with the notion that individuals (in the present case, heads of households) shift to less-risky personal investments as they age. Also worthy of note is the relative constancy, over the 30-year time span, of rates of return for each of the four age brackets.

## 4. Results

In this section we report point estimates and interval estimates of the intertemporal substitution elasticities for each cohort – first, under the assumption that households' utility functions are negative-exponential, and second, under the assumption that their utility functions are of the extended power type. We obtained point estimates by fitting the sample-mean values of consumption, income, and net worth directly to the equations of the optimization model. The details of this "fitting" procedure differ from one utility function to the other, and are explained below.

Our interval estimates account for sampling error in the household surveys administered by the BLS and by the Federal Reserve. Using the sample-mean values of consumption, income, and net worth together with their respective standard errors, we simulated 10,000 observations for each cohort. In generating these values, we treated the distribution of each sample-mean as normal, in accordance with the central limit theorem; in addition, we assumed that those distributions are independent of one another. The variation in sample-means feeds through to the projections on net worth and to the final estimates of net worth, so that each of the 10,000 simulated observations features values of those magnitudes that differ, at least modestly, from the values reported in Tables 3 and 4. In turn, for each observation, the simulated values of consumption, income, and net worth are fitted to the equations of the optimization model, with the result that each observation produces its own set of elasticity values. Based on the distribution of simulated elasticity values, we report 95 percent confidence intervals, the endpoints of which are the 2.5% and 97.5% quantiles of those distributions.

### *4.1. Results for negative-exponential utility*

Assume the negative-exponential function (14) accurately depicts household preferences. When, for one or the other of the two cohorts, we insert the relevant data-values $\bar{c}_1$, $\bar{c}_2$, and $\bar{c}_3$ (from Table 1), along with the data-values $\overline{R}_2$ and $\overline{R}_3$ (from Table 5) into (15), we have two equations in two unknowns, $\theta$ and $\beta$. Provided a solution exists, we thereby obtain point estimates of the true utility parameter values. For the utility function at hand, we denote those estimates by $\hat{\theta}$ and $\hat{\beta}$ :

$$\hat{\theta} = \frac{\ln(\overline{R}_2/\overline{R}_3)}{2\bar{c}_2 - \bar{c}_1 - \bar{c}_3}, \tag{24}$$

$$\hat{\beta} = \frac{R_2^{(\bar{c}_3-\bar{c}_2)/(2\bar{c}_2-\bar{c}_1-\bar{c}_3)}}{R_3^{(\bar{c}_2-\bar{c}_1)/(2\bar{c}_2-\bar{c}_1-\bar{c}_3)}}. \tag{25}$$

Using the data-values that apply to Cohort I, we obtain $\hat{\theta} = 7.04133 \times 10^{-6}$ and $\hat{\beta} = 0.40840$. Solution of the three-period utility-maximization problem, using the values of $\hat{\theta}$ and $\hat{\beta}$ along with the data-values of income, assets, and rates of return, yields optimal consumption values that match the data-values; that is, $(c_1^*, c_2^*, c_3^*) = (\bar{c}_1, \bar{c}_2, \bar{c}_3)$. Recalling that $\sigma_t = \theta c_t (t = 1, 2, 3)$ for the negative-exponential utility function, we have all values that are necessary to obtain point estimates of the substitution elasticities, which we denote by $\hat{\varepsilon}_{ij}$; see Table 6, in which we also report confidence intervals, based upon the simulation procedure described above, for each of the elasticities.

For each of the two cohorts, our confidence intervals suggest that $\varepsilon_{22}$ and $\varepsilon_{33}$ are both positive. For Cohort I, we have $\hat{\varepsilon}_{22} = 0.394$, with a 95 percent confidence interval of (0.174, 0.588), and $\hat{\varepsilon}_{33} = 0.554$, with a 95 percent confidence interval of (0.242, 0.826). For Cohort II, we have $\hat{\varepsilon}_{22} = 0.440$, with a 95 percent confidence interval of (0.239, 0.700), and $\hat{\varepsilon}_{33} = 0.492$, with a 95 percent confidence interval of (0.267, 0.771). These results suggest (with due caution) that, for both cohorts, $\varepsilon_{33} > \varepsilon_{22}$. That is, for both cohorts, our point estimates $\hat{\varepsilon}_{22}$ and $\hat{\varepsilon}_{33}$ suggest that the response of the ratio $c_3/c_2$ to a change in $R_3$, appears to be stronger than the response of the ratio $c_2/c_1$ to a change in $R_2$. This is particularly true for Cohort I, members of which are becoming elderly during their third period of life. These results are sensible in that they suggest that older people have a greater tendency to respond to higher interest rates by postponing present consumption in favor of future consumption, with declining levels of income looming. The estimates of the cross-period elasticities are sensible as well. For Cohort I, the cross-period elasticities are all positive, suggesting again that members of the older cohort desire to take advantage of higher inter-

Table 6
Estimates of utility-parameter values and elasticities: exponential utility function

| Utility-parameter values | | Intra-period elasticities | | Cross-period elasticities | | | |
|---|---|---|---|---|---|---|---|
| $\hat{\theta}$ | $\hat{\beta}$ | $\hat{\epsilon}_{22}$ | $\hat{\epsilon}_{33}$ | $\hat{\epsilon}_{21}$ | $\hat{\epsilon}_{23}$ | $\hat{\epsilon}_{31}$ | $\hat{\epsilon}_{32}$ |
| *Cohort I (age 45 in 1977)* | | | | | | | |
| $5.042\times10^{-6}$ | 0.4084 | 0.394 (0.174, 0.588) | 0.554 (0.242, 0.826) | 0.028 (0.005, 0.051) | 0.019 (0.003, 0.040) | 0.115 (0.089, 0.139) | 0.153 (0.094, 0.211) |
| *Cohort II (age 35 in 1977)* | | | | | | | |
| $7.041\times10^{-6}$ | 0.4593 | 0.440 (0.239, 0.700) | 0.492 (0.267, 0.771) | −0.032 (−0.046, −0.018) | −0.021 (−0.027,−0.013) | 0.021 (0.007, 0.034) | 0.035 (0.010, 0.068) |

Note: Single numbers represent point estimates; numbers in parentheses represent 95 percent confidence intervals.

est rates by smoothing consumption as the years of declining income are approached and realized. For Cohort I, the values of $\hat{\varepsilon}_{31}$ and $\hat{\varepsilon}_{32}$, while not especially large, are larger than the other cross-period elasticities, implying that, for members of that cohort, the ratio $c_3/c_2$ responds relatively strongly to increases in both $R_1$ and $R_2$.

It may be argued that, under perfect foresight, older consumers' intertemporal consumption ratios should be no more (or less) sensitive to interest rate changes than those of their younger counterparts. Indeed, uncertainty impacts the data-values we observe, and our optimization model abstracts from uncertainty. It is interesting, however, that the results reported here are consistent with some uncertainty-based models in which rational consumers postpone saving for retirement until relatively late in life. For example, Carroll and Samwick (1997) employed a buffer-stock model of saving to show that rational consumers, facing uncertain future incomes, begin to save for retirement only around age 50.

### 4.2. Results for extended power utility

Now assume the extended power utility function (19) accurately depicts household preferences. In this case, there are three utility-parameters – $\alpha$, $\gamma$, and $\beta$ – and only two equations in (20). We will show that, for a rather wide interval of $\alpha$-values, there exist corresponding values of $\gamma$ and $\beta$ that match the optimal values of consumption $(c_1^{**}, c_2^{**}, c_3^{**})$ with the data-values $(\bar{c}_1, \bar{c}_2, \bar{c}_3)$. We denote any set of parameter values that provides such a match by $\{\tilde{\alpha}, \tilde{\gamma}, \tilde{\beta}\}$. From (20) it follows that

$$\tilde{\gamma} = \frac{\ln(\overline{R}_3/\overline{R}_2)}{\ln[(\tilde{\alpha}+\bar{c}_3)/(\tilde{\alpha}+\bar{c}_2)] - \ln[(\tilde{\alpha}+\bar{c}_2)/(\tilde{\alpha}+\bar{c}_1)]}, \tag{26}$$

$$\tilde{\beta} = \exp\left(\frac{\ln\overline{R}_3 \ln[(\tilde{\alpha}+\bar{c}_2)/(\tilde{\alpha}+\bar{c}_1)] - \ln\overline{R}_2 \ln[(\tilde{\alpha}+\bar{c}_3)/(\tilde{\alpha}+\bar{c}_2)]}{\ln[(\tilde{\alpha}+\bar{c}_3)/(\tilde{\alpha}+\bar{c}_2)] - \ln[(\tilde{\alpha}+\bar{c}_2)/(\tilde{\alpha}+\bar{c}_1)]}\right). \tag{27}$$

For Cohort I, Table 7 displays nine sets of utility-parameter values that satisfy (26) and (27), along with the point estimates and confidence intervals that correspond to each set of parameter values. Noting that the smallest $\bar{c}_t$-value for Cohort I is \$298,390, the range of feasible $\tilde{\alpha}$-values is from −298,390 to positive infinity. As $\tilde{\alpha}$ approaches −298,390, $\tilde{\gamma}$

Table 7
Estimates of parameter values and elasticities: extended power utility function – Cohort I (age 45 in 1977)

| Utility-parameter values | | | Intra-period elasticities | | Cross-period elasticities | | | |
|---|---|---|---|---|---|---|---|---|
| $\alpha$ | $\tilde{\gamma}$ | $\tilde{\beta}$ | $\tilde{\epsilon}_{22}$ | $\tilde{\epsilon}_{33}$ | $\tilde{\epsilon}_{21}$ | $\tilde{\epsilon}_{23}$ | $\tilde{\epsilon}_{31}$ | $\tilde{\epsilon}_{32}$ |
| −250,000 | 0.4741 | 0.4446 | 0.678 | 0.174 | −0.056 | −0.040 | −0.234 | −0.438 |
| | | | (0.328, 0.990) | (0.043, 0.302) | (−0.104, −0.011) | (−0.079, −0.007) | (−0.279, −0.185) | (−0.602, −0.271) |
| −200,000 | 0.8223 | 0.4348 | 0.552 | 0.308 | −0.032 | −0.022 | −0.133 | −0.219 |
| | | | (0.254, 0.813) | (0.127, 0.473) | (−0.059, −0.006) | (−0.045, −0.004) | (−0.159, −0.105) | (−0.299, −0.136) |
| −100,000 | 1.519 | 0.4256 | 0.478 | 0.409 | −0.010 | −0.007 | −0.042 | −0.063 |
| | | | (0.213, 0.709) | (0.178, 0.613) | (−0.019, −0.002) | (−0.014, −0.001) | (−0.051, −0.003) | (−0.086, −0.039) |
| −10,000 | 2.1483 | 0.4216 | 0.453 | 0.448 | −0.000 | −0.001 | −0.003 | −0.005 |
| | | | (0.199, 0.675) | (0.197, 0.667) | (−0.001, −0.000) | (−0.001, −0.000) | (−0.004, −0.002) | (−0.006, −0.003) |
| 0 | 2.2183 | 0.4212 | 0.451 | 0.451 | 0 | 0 | 0 | 0 |
| | | | (0.198, 0.672) | (0.198, 0.672) | | | | |
| 10,000 | 2.2884 | 0.4209 | 0.449 | 0.454 | 0.001 | 0.000 | 0.003 | 0.004 |
| | | | (0.197, 0.669) | (0.200, 0.676) | (0.000, 0.001) | (0.000, 0.001) | (0.002, 0.004) | (0.004, 0.006) |
| 100,000 | 2.9194 | 0.4186 | 0.437 | 0.474 | 0.006 | 0.004 | 0.024 | 0.035 |
| | | | (0.192, 0.651) | (0.209, 0.705) | (0.001, 0.011) | (0.001, 0.008) | (0.019, 0.029) | (0.021, 0.047) |
| 1,000,000 | 9.2481 | 0.4120 | 0.407 | 0.527 | 0.020 | 0.014 | 0.084 | 0.114 |
| | | | (0.180, 0.609) | (0.231, 0.785) | (0.004, 0.037) | (0.002, 0.029) | (0.065, 0.101) | (0.070, 0.157) |
| $\infty$ | $\infty$ | 0.4084 | 0.394 | 0.554 | 0.028 | 0.019 | 0.115 | 0.153 |

Note: Single numbers represent point estimates; numbers in parentheses represent 95 percent confidence intervals.

approaches zero; as $\tilde{\alpha}$ approaches infinity, $\tilde{\gamma}$ approaches infinity as well (albeit more slowly than $\tilde{\alpha}$). It is reasonable to regard the case of $\tilde{\alpha} = 0$ – that is, the case of "narrow power utility" or constant-elasticity of marginal utility – as the baseline case, since it is assumed in most of the existing IES literature.

As is evident from Table 7, we can assert with 95 percent confidence – just as we could under the assumption of negative-exponential utility – that, if preferences are representable by the extended power utility function, the values of the intra-period elasticities $\varepsilon_{22}$ and $\varepsilon_{33}$ are positive and less than unity. In the baseline case, the implied common value of the intra-period elasticities is 0.451 for Cohort I, with a confidence interval of (0.198, 0.672). As we depart from the baseline case toward negative $\alpha$-values, $\tilde{\epsilon}_{22}$ increases and $\tilde{\epsilon}_{33}$ decreases, while the cross-period elasticities become negative and increase in absolute value. In contrast, as we depart from the baseline case toward positive $\alpha$-values, $\tilde{\epsilon}_{22}$ decreases and $\tilde{\epsilon}_{33}$ increases, while the cross-period elasticities become positive. If we ignore, or at least de-emphasize, those cases in which $\alpha$ is close to its minimum feasible value, we are left with a range of values for each of the elasticity point-estimates that is not terribly wide. For all relevant values of $\alpha$, our results strongly support the notion that the intra-period IES is positive for Cohort I households.

Table 8 displays results for Cohort II, for which the range of feasible $\tilde{\alpha}$-values is from $-400{,}460$ to positive infinity. In the baseline case ($\alpha = 0$) case, the intra-period elasticities have a common value of 0.469, with a confidence interval of (0.256, 0.738). Again, we observe $\tilde{\epsilon}_{22} > \tilde{\epsilon}_{33}$ for $\alpha < 0$ and $\tilde{\epsilon}_{22} < \tilde{\epsilon}_{33}$ for $\alpha > 0$. Ignoring those values of $\alpha$ that are negative and very large in absolute value, it is again the case that what remains is a rather narrow confidence interval for each of the elasticities. Furthermore, we can assert with a high degree of confidence that the intra-period IES is positive for Cohort II households as well as for Cohort I households.

For Cohort I, all of the cross-period elasticity estimates are negative when $\alpha$ is set to a negative value; they are all positive when $\alpha$ is set to a positive value. For Cohort II, the results are mixed. For both cohorts, $\alpha < 0$ implies $\epsilon_{31} < 0$ and $\epsilon_{32} < 0$, implying an increase in either $R_1$ or $R_2$ leads to a *decrease* in $c_3/c_2$. Thus, an assumption of $\alpha < 0$ implies something along the lines of dissaving in anticipation of retirement. Furthermore, $\alpha < 0$ implies that $\sigma$, the elasticity of marginal utility, increases with consumption, and that assumption is at odds with prevailing theoretical views. Hence, we regard the cases in which $\alpha \geqslant 0$ case as the more plausible ones.

Note that, for each of the two cohorts, as $\alpha$ approaches infinity, all of the elasticity point estimates (as well as the corresponding values of $\tilde{\beta}$) converge to the estimates that were derived under the assumption of negative-exponential utility. This result is not a coincidence; rather, we show in the Appendix that this convergence occurs in all cases, irrespective of the values from the data.

### *4.3. Implied annual subjective discount rates*

In view of the fact that the point estimate of $\beta$ represents an estimate of the gross 10-year discount *factor*, we have, for the estimated net annual discount *rate*, $1/\beta^{0.1} - 1$. For the negative exponential utility function, the point estimate of the net annual discount rate is 0.094 for Cohort I and 0.081 for Cohort II. For the extended power utility function, similar values are obtained for all values of $\alpha$, as the estimated value of $\beta$ varies relatively slightly as $\alpha$ varies over its feasible range; in the baseline case ($\alpha = 0$), we have, for Cohort

Table 8
Estimates of parameter values and elasticities: extended power utility function – Cohort II (Age 35 in 1977)

| Utility-parameter values | | | Intra-period elasticities | | Cross-period elasticities | | | |
|---|---|---|---|---|---|---|---|---|
| $\alpha$ | $\gamma$ | $\tilde{\beta}$ | $\tilde{\epsilon}_{22}$ | $\tilde{\epsilon}_{33}$ | $\tilde{\epsilon}_{21}$ | $\tilde{\epsilon}_{23}$ | $\tilde{\epsilon}_{31}$ | $\tilde{\epsilon}_{32}$ |
| −350,000 | 0.3609 | 0.4634 | 0.615 | 0.353 | 0.1837 | 0.121 | −0.116 | −0.169 |
| | | | (0.345, 0.922) | (0.179, 0.643) | (0.098, 0.275) | (0.069, 0.173) | (−0.192, −0.039) | (−0.306, −0.050) |
| −200,000 | 1.1230 | 0.4605 | 0.495 | 0.448 | 0.031 | 0.020 | −0.019 | −0.031 |
| | | | (0.271, 0.774) | (0.245, 0.715) | (0.017, 0.044) | (0.012, 0.026) | (−0.032, −0.007) | (−0.059, −0.009) |
| −100,000 | 1.6281 | 0.4601 | 0.478 | 0.462 | 0.010 | 0.007 | −0.007 | −0.011 |
| | | | (0.261, 0.751) | (0.253, 0.730) | (0.006, 0.015) | (0.004, 0.009) | (−0.011, −0.002) | (−0.021, −0.003) |
| −10,000 | 2.0823 | 0.4600 | 0.470 | 0.468 | 0.001 | 0.001 | −0.001 | −0.001 |
| | | | (0.256, 0.739) | (0.256, 0.737) | (0.000, 0.001) | (0.000, 0.001) | (−0.001, 0.000) | (−0.002, 0.000) |
| 0 | 2.1327 | 0.4600 | 0.469 | 0.469 | 0 | 0 | 0 | 0 |
| | | | (0.256, 0.738) | (0.256, 0.738) | | | | |
| 10,000 | 2.1832 | 0.4599 | 0.468 | 0.469 | −0.001 | −0.001 | 0.000 | 0.001 |
| | | | (0.255, 0.737) | (0.256, 0.739) | (−0.001, 0.000) | (−0.001, 0.000) | (0.000, 0.001) | (0.000, 0.002) |
| 100,000 | 2.6372 | 0.4598 | 0.463 | 0.473 | −0.006 | −0.004 | 0.004 | 0.007 |
| | | | (0.253, 0.730) | (0.258, 0.745) | (−0.009, −0.004) | (−0.005, −0.002) | (0.001, 0.007) | (0.002, 0.013) |
| 1,000,000 | 7.1759 | 0.4595 | 0.448 | 0.485 | −0.023 | −0.015 | 0.015 | 0.025 |
| | | | (0.244, 0.711) | (0.264, 0.762) | (−0.033, −0.013) | (−0.019, −0.009) | (0.005, 0.024) | (0.007, 0.048) |
| ∞ | ∞ | 0.4593 | 0.440 | 0.492 | −0.032 | −0.021 | 0.021 | 0.035 |

Note: Single numbers represent point estimates; numbers in parentheses represent 95 percent confidence intervals.

I, a point estimate of beta of 0.090, while for Cohort II that estimate is 0.081. These values are not out of line with estimates from the existing literature. While parameterizations of real business cycle models typically employ an annual discount rate of 2 percent to 3 percent, Gourinchas and Parker (2002) obtained point estimates of 4.0–4.5%. Carroll and Samwick's (1997) baseline case produced a point estimate of 10.7%, with higher values for relatively small deviations in values of other parameters.

## 5. Summary and conclusion

In our study reported upon here, we departed from the methods that Hall (1988) and many other authors have employed for the purpose of estimating the intertemporal elasticity of substitution. Our departure was twofold. Rather than employing regression analysis on aggregate consumption data, we posited a general life-cycle model as a means of capturing differences in income, consumption, and asset accumulation among consumers in different age brackets. In addition, our approach allowed us to experiment with different utility specifications, whereas previous efforts have relied almost exclusively upon the narrow power (isoelastic) function.

The application of survey data to our model produced estimates of the intra-period IES that are strongly suggestive of a positive value for that magnitude. The consistency of this result over two cohorts of households and its robustness with respect to alternative utility specifications lends an important sense of credibility to our approach. Our evidence suggests, roughly, that intra-period substitution elasticities (which are the focus of the existing literature) are probably in the neighborhood of 0.2–0.8. Cross-period elasticities are probably relatively small, but further investigation of their values seems desirable.

References to "the" intertemporal elasticity of substitution fail to capture the possibility that the strength of the response of intertemporal consumption ratios to changes in interest rates may change over time, particularly over long periods of time. As the age structure of a population changes, intertemporal elasticities are likely to change as well, and we can attribute changes in the IES over time to at least two distinct sources. First, over a given period of time, households headed by young persons experience different levels of income, consumption, and assets when compared with households headed by older persons, and the value of the IES may well be sensitive to those differences. A second source of change is intergenerational changes in preferences, that is, changes in utility-parameters. The effects of macroeconomic policies have been shown to depend, in an important way, on the relative strength of intertemporal substitution. Attempts such as ours to relate that relative strength to demographic characteristics warrant further study.

## Appendix

Let $\hat{\varepsilon}_{ij}$ and $\tilde{\varepsilon}_{ij}$ denote, respectively, the implied substitution elasticities for the negative exponential utility function and for the extended power utility function; similarly, let $\hat{\sigma}$ and $\tilde{\sigma}$ denote the respective elasticities of marginal utility. From (7)–(12) it is clear that, using the same data-values of consumption, interest rates, and assets in all cases, $\hat{\varepsilon}_{ij} = \tilde{\varepsilon}_{ij}$ if $\hat{\sigma}_t = \tilde{\sigma}_t$. For the negative-exponential case, $\hat{\sigma}_t = \hat{\theta}\bar{c}$, and it follows from (23) that

$$\hat{\sigma}_t = \frac{\bar{c}_t \ln(\overline{R}_2/\overline{R}_3)}{2\bar{c}_2 - \bar{c}_1 - \bar{c}_3}. \tag{A1}$$

For the extended power case, $\tilde{\sigma}_t = \tilde{\gamma}\bar{c}_t/(\tilde{\alpha} + \bar{c}_t)$, and it follows from (25) that

$$\tilde{\sigma}_t = \frac{\bar{c}_t \ln(\overline{R}_3/\overline{R}_2)}{(\tilde{\alpha} + \bar{c}_t)\{\ln[(\tilde{\alpha} + \bar{c}_3)/(\tilde{\alpha} + \bar{c}_2)] - \ln[(\tilde{\alpha} + \bar{c}_2)/(\tilde{\alpha} + \bar{c}_1)]\}}. \tag{A2}$$

Rewrite the denominator of (A2) as

$$\frac{f(\tilde{\alpha})}{g(\tilde{\alpha})} = \frac{\ln[(\tilde{\alpha} + \bar{c}_3)/(\tilde{\alpha} + \bar{c}_2)] - \ln[(\tilde{\alpha} + \bar{c}_2)/(\tilde{\alpha} + \bar{c}_1)]}{1/(\tilde{\alpha} + \bar{c}_t)}. \tag{A3}$$

We have

$$\frac{f'(\tilde{\alpha})}{g'(\tilde{\alpha})} = \frac{(\tilde{\alpha} + \bar{c}_t)^2[(\tilde{\alpha} + \bar{c}_1)(\bar{c}_3 - \bar{c}_2) - (\tilde{\alpha} + \bar{c}_3)(\bar{c}_2 - \bar{c}_1)]}{(\tilde{\alpha} + \bar{c}_1)(\tilde{\alpha} + \bar{c}_2)(\tilde{\alpha} + \bar{c}_3)}. \tag{A4}$$

The numerator and denominator of (A4) are both cubic equations in $\tilde{\alpha}$. In the numerator, the coefficient on the cubic term is $2\bar{c}_2 - \bar{c}_1 - \bar{c}_3$, while the coefficient on the cubic term in the denominator is 1. Using these results and L'Hospital's rule, it follows that

$$\lim_{\tilde{\alpha}\to\infty}\left(\frac{f(\tilde{\alpha})}{g(\tilde{\alpha})}\right) = \lim_{\tilde{\alpha}\to\infty}\left(\frac{f'(\tilde{\alpha})}{g'(\tilde{\alpha})}\right) = 2\bar{c}_2 - \bar{c}_1 - \bar{c}_3. \tag{A5}$$

In turn it follows from (A1)–(A5) that

$$\lim_{\tilde{\alpha}\to\infty} \tilde{\sigma}_t = \hat{\sigma}_t. \tag{A6}$$

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